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Solving Fuzzy Maximal Flow Problem Using Octagonal Fuzzy Number

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Abstract

In this paper a general fuzzy maximal flow problem is discussed . A crisp maximal flow problem can be solved in two methods : linear programming modeling and maximal flow algorithm . Here I tried to fuzzify the maximal flow algorithm using octagonal fuzzy numbers introduced by S.U Malini .C. kennedy [26]. By ranking the octagonal fuzzy numbers it is possible to compare them and using this we convert the fuzzy valued maximal flow algorithm to a crisp valued algorithm . It is proved that a better solution is obtained when it is solved using fuzzy octagonal number than when it is solved using trapezoidal fuzzy number . To illustrate this a numerical example is solved and the obtained result is compared with the existing results . If there is no uncertainty about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems.

Keywords: Fuzzy maximal flow problem , ranking functions , Octagonal fuzzy Number

I.   Introduction

In optimization theory maximum flow problems involve finding a feasible amount of flow passing from a source to a sink which is maximum. It is a special case of more complex network flow problems such as circulation problems, communication networks, oil pipeline systems, power systems and so on. The maximum flow problem was first formulated in 1954 by T .E Harris and F.S Ross [17] as a simplified model of Soviet railway traffic flow. In 1955 Lester .R. Ford jr and Delbert R Fulkerson [15] crated the first known augmenting path algorithm. Later on various improved solutions to the maximum flow algorithm were discovered. Some of them are, the shortest augmenting path algorithm of Edmonds and Karp, the blocking flow algorithm of Dinitz , the push relabel algorithm of Goldberg and Tarjan and the binary blocking algorithm of Goldberg and Rao . The electrical flow algorithm of Christiano , kelner ,Mardy and Spielman is useful in finding an approximately optimal maximal flow of an undirected graph . These are some of the efficient methods to solve crisp maximal flow algorithms [19].

In conventional maximal flow problems, it is assumed that the decision maker is certain about the flows between the different nodes . But in real life situations there always exist uncertainty about the parameters such as cost , capacities and demand of maximal flow problem . In such situations flows may be represented as fuzzy numbers . The problem of finding the maximum flow between a source and a sink with fuzzy capacities has a wide range of applications in communication networks, oil pipeline systems , power systems etc .

There are only few number of papers published dealing with fuzzy maximal flow problems .The paper " Fuzzy Flows on Networks "by K.Kim and Roush [22] is considered as one of the first papers on this subject . The authors used fuzzy matrices to obtain a fuzzy optimal flow.

Chanas and Kolodziejczyk approached this problem using minimum cuts technique. They presented an algorithm [4] in 1982 for a graph with crisp structure and fuzzy capacities i.e , the arcs have a membership function associated in their flow .Later in 1984 they studied this problem in another way in which the flow is a real number and the capacities have upper and lower bounds with a satisfaction function [5] .In 1986 Chanas and Kolodziejczyk [6] had also studied the integer flow and proposed an algorithm . Chanas et al. [3] (1995) studied the maximum flow problem when the underlying associated structure is not well defined and must be modeled as a fuzzy graph. Diamond(2001) developed interval-valued versions of the max-flow min cut theorem and Karp-Edmonds algorithm and provide robustness estimates for flows in networks in an imprecise or uncertain environment[14]. These results are extended to networks with fuzzy capacities and flows.

Liu and Kao (2004) [24] investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Ji et al.(2006) [20] considered a generalized fuzzy version of maximum flow problem, in which arc capacities are fuzzy variables. Hernandes et al. (2007) [18] proposed an algorithm, based on the classic algorithm of Ford-Fulkerson. The algorithm uses the technique of the incremental graph and representing all the

parameters as fuzzy numbers. Kumar et al. (2009) [23] proposed a new algorithm to find fuzzy maximal flow between source and sink by using ranking function.

In this paper an algorithm is given to find the fuzzy maximal flow between the source and the sink for a directed graph by representing all the parameters as octagonal fuzzy numbers . To illustrate the algorithm a numerical example is solved and the obtained result is compared with the existing result . Also the numerical problem is converted to one in which all the parameters are taken as trapezoidal fuzzy numbers and both the results are compared . A ranking using α-cut is introduced on octagonal fuzzy numbers. Using this ranking the fuzzy maximal flow problem is converted to a crisp valued problem, which can be solved using maximal flow algorithm. The optimal solution can be got either as a fuzzy number or as a crisp number

II.   Octagonal Fuzzy Numbers : Basic Definitions

Octagonal fuzzy numbers are proposed by Malini .S.U and kennedy Felbin .C in 2013 [26] [27] . We recall the required definitions and results .

Definition 2.1:

An octagonal fuzzy number denoted by $\tilde{A}_\omega$ is defined to be the ordered quadruple $\tilde{A}_\omega = \left(l_1(r), s_1(t), s_2(t), l_2(r)\right)$ for $r \in [0,k]$ and $t \in [k, \omega]$ where

1. $l_1(r)$ is a bounded left continuous non decreasing function over $[0, \omega_1]$, $0 \le \omega_1 \le k$
2. $s_1(t)$ is a bounded left continuous non decreasing function over $[k, \omega_2]$, $k \le \omega_2 \le \omega$
3. $s_2(t)$ is a bounded left continuous non increasing function over $[k, \omega_2]$, $k \le \omega_2 \le \omega$
4. $l_2(r)$ is a bounded left continuous non increasing function over $[0, \omega_1]$, $0 \le \omega_1 \le k$

Remark 2.1 :

If ω =1 then the above defined number is called a normal octagonal fuzzy number .

Definition 2.2 :

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by   ($a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8,$) where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for } x \le a_1 \\[2mm] k\left(\frac{x-a_1}{a_2-a_1}\right) & \text{for } a_1 \le x \le a_2 \\[2mm] k & \text{for } a_2 \le x \le a_3 \\[2mm] k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & \text{for } a_3 \le x \le a_4 \\[2mm] 1 & \text{for } a_4 \le x \le a_5 \\[2mm] k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & \text{for } a_5 \le x \le a_6 \\[2mm] k & \text{for } a_6 \le x \le a_7 \\[2mm] k\left(\frac{a_8-x}{a_8-a_7}\right) & \text{for } a_7 \le x \le a_8 \\[2mm] 0, & \text{for } x \ge a_8 \end{cases}$$

Where $0 \le k \le 1$

Remark 2.2:

If k=0 , the octagonal fuzzy number reduces to trapezoidal fuzzy number (a_3, a_4, a_5, a_6) and if k =1 it reduces to trapezoidal fuzzy number (a_1, a_4, a_5, a_8)
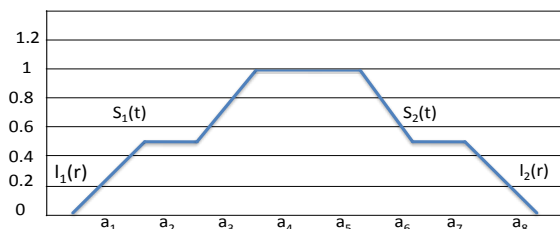
Remark 2.3 : According to the above mentioned definition , octagonal fuzzy number $\breve{A}_\omega$ is the ordered quadruple $\left(l_1(r), s_1(t), s_2(t), l_2(r)\right)$ where $r \in [0, \ k]$ and $t \in [k, \ \omega]$ where

$l_1(r) = k\left(\frac{r-a_1}{a_2-a_1}\right) s_1(r) = k + (1-k)\left(\frac{t-a_3}{a_4-a_3}\right) s_2(r) = k + (1-k)\left(\frac{a_6-t}{a_6-a_5}\right)$ and $l_2(r) = k\left(\frac{a_8-r}{a_8-a_7}\right)$

Remark 2.4 :
Membership function $\mu_{\tilde{A}}(x)$ are continuous functions
Graphical representation of an octagonal fuzzy number for k = 0.5 is

Remark 2.5:

If $\tilde{A}$ be an octagonal fuzzy number, then the α – cut of $\tilde{A}$ is

$$[\breve{A}]_\alpha = \{x/\breve{A}(x) \geq \alpha\}$$

$$= \begin{cases} [l_1(\alpha), l_2(\alpha)] & for\ \alpha \in [0, k) \\ \\ [s_1(\alpha), s_2(\alpha)] & for\ \alpha \in [k, 1] \end{cases}$$

The octagonal fuzzy number is convex as their α – cuts are convex sets in the classical sense.

Remark 2.6:
The collection of all octagonal fuzzy numbers from $\mathcal{R}$ to I is denoted by $\mathcal{R}\omega(I)$ and if ω =1 the collection of all normal octagonal fuzzy number is denoted by $\mathcal{R}(I)$.

Remark 2.7:
A fuzzy number is called positive (negative) , denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$)if its membership function $\mu_{\tilde{A}}(x)$ satisfie $\mu_{\tilde{A}}(x) = 0 \ \forall\ x \leq 0$ ($\forall x \geq 0$)

Remark 2.8:
Two octagonal fuzzy numbers $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ & $\breve{B} = (b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8)$ are said to be equal if and only if $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$, $a_5=b_5$, $a_6=b_6$, $a_7=b_7, a_8=b_8$.

Remark 2.9 :
Using interval arithmetic given by Kaufmann [21] .A we obtain α – cuts , addition , subtraction and multiplication of two octagonal fuzzy numbers as follows

- α – cut of an octagonal fuzzy number : To find the α – cut of a normal octagonal fuzzy number $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ for α∈[0,1]

$$[\tilde{A}]_\alpha = \begin{cases} [a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)] & for\ \alpha \in [0, k] \\ \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)] & for\ \alpha \in [k, 1] \end{cases}$$

- Addition of octagonal fuzzy numbers : Let $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ & $\breve{B} = (b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8)$ be two octagonal fuzzy numbers . We add the α – cuts of $\tilde{A}$ and $\breve{B}$ using interval arithmetic

$$[\tilde{A}]_\alpha + [\tilde{B}]_\alpha = \begin{cases} [a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)] \\ +[b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1), b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)] \quad for\ \alpha \in [0, k] \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)] \\ +[ba_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)] \quad for\ \alpha \in [k, 1] \end{cases}$$

- Subtraction of two octagonal fuzzy numbers :

Let $\breve{A}$ = ($a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8$) & $\tilde{B}$ = ($b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8$) be two octagonal fuzzy numbers . We subtract the α – cuts of $\breve{A}$ and $\tilde{B}$ using interval arithmetic

$$[\tilde{A}]_\alpha - [\tilde{B}]_\alpha = \begin{cases} [a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)] \\ -[b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1), b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)] \quad for\ \alpha \in [0, k] \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)] \\ -[ba_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)] \quad for\ \alpha \in [k, 1] \end{cases}$$

- Fuzzy Marix : A matrix $\breve{A} = (\tilde{a}_{ij})$ if each element of $\breve{A}$ is a fuzzy number . A fuzzy matrix $\breve{A}$ will be positive and denoted by $\breve{A} > 0$ if each element of $\breve{A}$ is positive .
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III.   Ranking of fuzzy octagonal numbers

Fuzzy maximal flow problem have received much attention in the recent years .Yager's ranking method [32] is one of the robust ranking techniques which is used to solve fuzzy maximal flow problems involving triangular and trapezoidal numbers .A similar measure for a fuzzy octagonal number is introduced by S.U Malini et al .In this paper this method is used to convert the octagonal fuzzy numbers to crisp values .

Definition 3.1 :
A measure of normal octagonal fuzzy number $\tilde{A}$ is a function $M_\alpha : \mathcal{R}(I) \to \mathcal{R}^+$ which assigns a non-negative real number $M_\alpha(\breve{A})$ that express the measure of $\tilde{A}$, $M_\alpha^{oct}(\breve{A}) = \frac{1}{2}\int_\alpha^k (l_1(r) + l_2(r))dr + \frac{1}{2}\int_k^1 (s_1(t) + s2tdt$ where 0< k < 1
Definition 3.2 :
Let $\tilde{A}$ be a normal octagonal fuzzy number . The value $M_\alpha^{oct}(\breve{A})$, called the measure of $\tilde{A}$ is calculated as follows :
$M_\alpha^{oct}(\breve{A}) = \frac{1}{4}\big((a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1 - k)\big)$

IV.   Mathematical Formulation of a fuzzy maximal Flow Problem

In this section , we consider a pipe network used to transfer fluid (oil, water etc) from one location to another . The maximum flow of fluid in each pipe will be limited to a particular value depending on the diameter or the slope of the pipe in that segment . We consider the pipe segment between any two locations i and j usually called as the arc from i – j of the network will have maximum flow of fluid per unit time from node i to node j which we are considering as a fuzzy number . We try to solve this problem to determine the maximum flow of liquid from a given source to a given destination using the fuzzy maximal flow algorithm

V.   Procedure to Solve Fuzzy Maximal Flow Problem (FMFP) algorithm

The objective of this problem is to find a maximal flow from a given source to a given destination using octagonal fuzzy number . The algorithm is as follows
Step 1:
Prepare a fuzzy capacity matrix $[\tilde{A}]$ for the given flow network. Let $(\tilde{a}_{ij})$ be the flow from the node i to the node j (i = 1,2,3,……n; j = 1,2,3,…n , where n is the total number of nodes) Set the iteration k = 1 , cumulative flow $\tilde{X} = 0$
Step 2:
Find a directed path starting from the given source to the given destination directly or passing through some intermediate nodes which will have some feasible quantity of flow.
Step 3:
 If such a path exist then go to next step , otherwise go to step 7

Step 4:
 Find the minimum of the capacities of the various paths traced in step 2 . Let it be $\tilde{Q}_k$. Set $\tilde{X} = \tilde{X} + \tilde{Q}_k$

Step 5:
 Obtain the next flow matrix as :
a)   Subtract $\tilde{Q}_k$ from all $\tilde{a}_{ij}$ values corresponding to forward path traced
b)   Add $\tilde{Q}_k$ to all $\tilde{a}_{ij}$ values corresponding to backward path traced

Step 6: Set k = k+1 and go to step 2
 Step 7:
Obtain a new flow matrix$\left[\tilde{C}\right]$ by subtracting the elements of the flow matrix $\left[\tilde{B}\right]$ in the last iteration from the corresponding elements of the starting fuzzy capacity matrix $\left[\tilde{A}\right]$

$$\tilde{c}_{ij} = \tilde{a}_{ij} - \tilde{b}_{ij} \ , if \ \tilde{a}_{ij} < \tilde{b}_{ij} \ \text{ for all values of i and j}$$
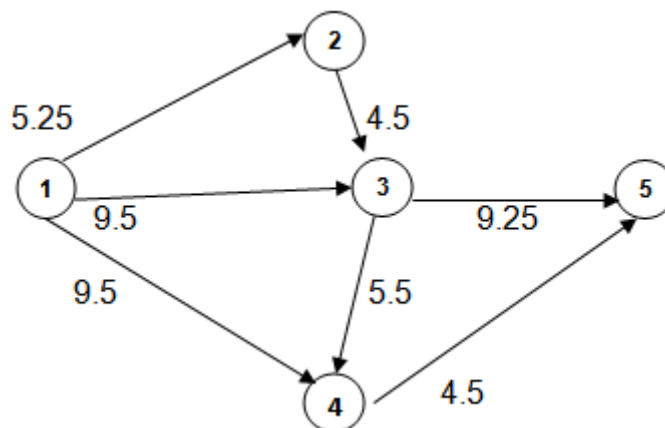$$= 0, \ \text{otherwise}$$

Step 8 :
a)   The cell entries of the fuzzy matrix $\left[\tilde{C}\right]$ represent the flows in various arcs .
b)   The maximal flow from the source node to the destination node is $\tilde{X}$
c)   Map the cell entries on to the corresponding arcs of the network showing the fuzzy flow s in various arcs to get the maximal flow $\tilde{X}$.

VI.   6. Illustrative Example .
        Consider the directed flow network shown below with fuzzy flow capacities between various pairs of location . Find the maximal flow from node 1 to node 5.

        First we convert the fuzzy values into the crisp values using ranking function which is also given in the following table

Node i-j	Fuzzy capacities	Crisp values using ranking function
1-2	(1,2,3,5,6,7,8,10)	5.25
1-3	(4,5,7,8,11,12,14,15)	9.5
1-4	(4,5,7,8,10,13,14,15)	9.5
2-3	(1,2,3,4,5,6,7,8)	4.5
3-4	(1,2,3,4,5,6,7,8,11)	5.5
3-5	(4,5,7,8,10,12,13,15)	9.25
4-5	(0,1,3,4,5,6,7,10)	4.5

Step 1: Fuzzy capacity Matrix using ranking function :

	1	2	3	4	5
1	—	5.25	9.5	9.5	0
2	0	—	4.5	0	0
3	0	0	—	5.5	9.25
4	0	0	0	—	4.5
5	0	0	0	0	—

First assume the path

1-2-3-5

let Q_1 = min (5.25,4.5,9.25)

$= 4.5$

$X = 4.5$

	1	2	3	4	5
1	—	0.75	9.5	9.5	0
2	4.5	—	0	0	0
3	0	4.5	—	5.5	4.75
4	0	0	0	—	4.5
5	0	0	4.5	0	—

Now assume the path 1-3-4-5

Q_2 = min (9.5,5.5,4.5)

$= 4.5$

$X = Q_2 + 4.5 = 9$

	1	2	3	4	5
1	—	0.75	5	9.5	0
2	4.5	—	0	0	0
3	4.5	4.5	—	1	4.75
4	0	0	4.5	—	0
5	0	0	4.5	4.5	—

Assume the path 1 —3 —5

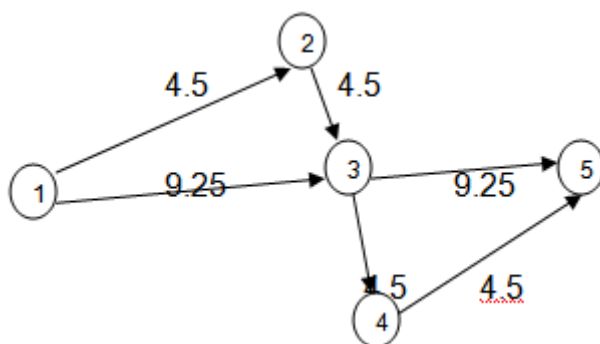Q_3 = min (5,4.75)

$= 4.75$

$X = Q_3 + 9$

$= 13.75$

Now no more flow is possible from node 1 to node 5

Final flow matrix

$$
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\left(
\begin{array}{ccccc}
- & 4.5 & 9.25 & 0 & 0 \\
0 & - & 4.5 & 0 & 0 \\
0 & 0 & - & 4.5 & 9.25 \\
0 & 0 & 0 & - & 4.5 \\
0 & 0 & 0 & 0 & -
\end{array}
\right)
\end{array}
$$

The maximal flow from node 1 to node 5 is 13.75

Hence by maximal flow algorithm the optimum solution is

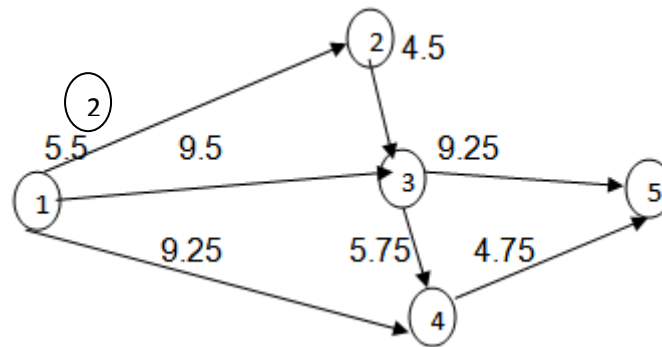| Node i-j	Fuzzy capacities	Crisp values using ranking function
1-2	(1,2,3,4,5,6,7,8)	4.5
1-3	(4,5,7,8,10,12,13,15)	9.25
2-3	(1,2,3,4,5,6,7,8)	4.5
3-4	(1,2,3,4,5,6,7,8)	4.5
3-5	(4,5,7,8,10,12,13,15)	9.25
4-5	(1,2,3,4,5,6,7,8)	4.5
Total flow at node 1	(5,7,10,12,15,18,20,23)	13.75

Remark 6.1:
So we are getting the same value when we assume that the network is having crisp and fuzzy capacities.
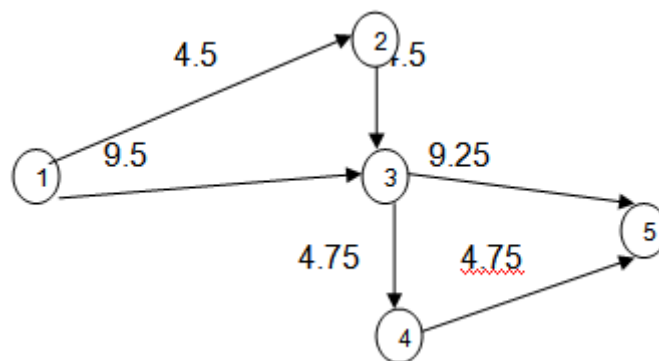
Remark 6.2:
When we convert the octagonal fuzzy maximal flow problem to trapezoidal fuzzy maximal flow problem

Node i-j	Fuzzy capacities	Crisp values using ranking function
1-2	(1,5,6,10)	5.5
1-3	(4, 8,11,15)	9.5
1-4	(4,8,10, 15)	9.25
2-3	(1,4,5,8)	4.5
3-4	(1,5,6,11)	5.75
3-5	(4,8,10,12,15)	9.25
4-5	(0,4,5,10)	4.75

When this problem is solved using the above algorithm the optimal solution is as follows

Node i-j	Fuzzy capacities	Crisp values using ranking function
1-2	(1,4,5,8)	4.5
1-3	(4, 8,11,15)	9.5
2-3	(1,4,5,8)	4.5
3-4	(0,4,5,10)	4.75
3-5	(4,8,10,12,15)	9.25
4-5	(0,4,5,10)	4.75
Total flow at node 1	(5,12,16,23)	14

The crisp value of the optimum fuzzy maximal flow and the corresponding fuzzy flow are the same . But it is slightly greater than the value obtained when octagonal fuzzy number is used .

VII. Conclusion

In this paper an algorithm is proposed for finding the fuzzy optimal flow of the fuzzy maximal flow problem using octagonal fuzzy number . The result is also compared using trapezoidal fuzzy number . A numerical example is solved to illustrate the proposed result . The concept of finding the fuzzy optimal solution of fuzzy maximal flow problems presented in this paper, is quite general in nature and can be extended to solve the other network flow problems like shortest path problems, critical path method etc

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